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Hipparcos catalogue orientation as obtained from observations of minor planets

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Abstract. Ground–based photographic observations of 12 minor planets obtained at Nikolaev observatory (Ukraine) between 1961–1995 were reduced to the Hipparcos catalogue system and processed by the least squares method (LSM) separately and in combination with the Hipparcos NDAC (the Northern Data Analysis Consortium) and FAST (the Fundamental Astronomy by Space Techniques) observations of 48 minor planets. The aim was to estimate the accuracy of the Nikolaev observations and to determine the orientation of the International Celestial Reference System (ICRS) with respect to the DE200/LE200 dynamic frame of reference. The Nikolaev observations proved to be sufficiently accurate, the unit weight error being of order of $0.15''$. The results of separate processing of Nikolaev and Hipparcos observations were not satisfactory for two reasons. First, the Nikolaev observations only measured accurately the changes of orientation parameters, and second, the Hipparcos observations only accurately measured the values of the orientation parameters themselves. However by combining the observations, the accuracy was greatly enhanced. The weight of the Hipparcos observations was taken to be unity and that one of the Nikolaev observations was chosen as 0.01, in accordance with the corresponding unit weight errors. The photocentre offsets were taken into account and three models of these offsets were considered. In the final processing the offset model based on the Lommel–Seeliger law of light scattering at the asteroid surface was used.

The values of the orientation parameters of the ICRS and their time changes were obtained for epoch JD 2448439.5 (in *mas* for ϵ and *mas/year* for ω). The combined solution based on NDAC and Nikolaev observations is $\epsilon_x = 2.5 \pm 1.3, \epsilon_y = -12.7 \pm 2.2, \epsilon_z = 1.4 \pm 3.3, \omega_x = 0.4 \pm 0.3, \omega_y = -0.7 \pm 0.3, \omega_z = -0.9 \pm 0.6$. The combined solution based on FAST and Nikolaev observations is $\epsilon_x = 3.8 \pm 1.7, \epsilon_y = -11.0 \pm 2.1, \epsilon_z = -3.6 \pm 3.2, \omega_x = 0.3 \pm 0.3, \omega_y = -0.6 \pm 0.3, \omega_z = -0.8 \pm 0.6$. The values obtained for ω are of the same order as their errors. The obtained estimates and those of Folkner et al. (1994) range in accordance with 3σ–tolerances.

Key words: astrometry – reference systems – minor planets, asteroids

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1. Introduction

Until recently, star catalogues were classified by means of a dynamical frame of reference, determined by the equator and the equinox of the epoch. This classification was first realised through the observations of the major planets and the Sun. The corresponding methods were developed by S.Newcomb (1895).

The idea of using minor planets for this purpose was first proposed by Dyson (1928). The plans for observing specially selected minor planets (SMP) for the catalogue were elaborated in 1933–1936 (Numerov 1933, 1935a, 1935b, 1935c, 1935d, 1936), (Brouwer 1935). B.Numerov (1933) proposed to observe 10 minor planets (1, 4, 40, 122, 124, 189, 196, 208, 214, 407) within the equatorial zone $\delta = \pm 10°$ during 8 to 10 year period. The list of minor planets in (Brouwer 1935) included the asteroids: 1, 2, 3, 4, 6, 7, 12, 25, 57, 185, 216, 287, 409, 532. In this paper preference was given to the minor planets having longer observation history, more noticeable brightness, larger mean motion and small eccentricity. B. Numerov (1935d, 1936) proposed a list of minor planets (2, 3, 6, 18, 39, 51, 105, 287, 354) to achieve a combined determination of minor planet and Earth element corrections. These plans were approved by the IAU at its 5th General Assembly in 1935. The work on the D.Brouwer's plan lasted until 1948. During WWII a restricted number of observatories participated in this work.

The work on B.Numerov's plan began after N.Samoilova–Yakhontova's report at the Astrometric conference in Poulkovo in 1954 (Samoilova–Yakhontova 1955). She revived and improved B.Numerov's ideas and plans and connected them with work on the Catalogue of the less bright stars. The delay of almost twenty years was caused by two factors: in 1936 B.Numerov was groundlessly put under arrest and died in prison in 1941; and it required some time for liquidation of the WWII aftermath in the Soviet Union. The list of minor planets in (Samoilova–Yakhontova 1955) included 10 asteroids: 1, 2, 3, 4, 6, 7, 11, 18, 39, 40.

Many observatories in the USSR and in the West participated in N.Samoilova–Yakhontova's program. The observations obtained have been processed (Pierce 1971), (Orelskaya 1978, 1980) and used for improving the zero–points of the FK5 catalogue (Fricke 1980, 1982).

V.Orelskaya (1975) proposed to extend the number of the SMP to 20 and add to N.Samoilova–Yakhontova's list of the SMP the following minor planets: 25, 148, 389, 480, 532, 568, 582, 594, 704, 1301. Her idea was to ensure good coverage of the $\delta = \pm 30°$ zone by observing the minor planets for 10 years. The idea was approved by the IAU Commissions 8 and 20 at the IAU General Assembly in 1976, and observations continued according to the new program. The results of this program were not fully satisfactory because some of the newly included minor planets had weak brightness and their observations were not sufficiently accurate for processing.

In 1991 the IAU approved the proposal of the Institute of Theoretical Astronomy of Russian Academy of Sciences (Leningrad) to continue the observations of minor planets up to 2000 and to shorten the list of the SMP to 15: 1, 2, 3, 4, 6, 7, 11, 18, 25, 39, 40, 148, 389, 532, 704 (Batrakov & Shor 1989); the weak brightness asteroids were dropped. The work on observing the SMP was regularly supported by issuing the daily ephemerides of the SMP (see, for example Batrakov et al. 1998c).

Some fundamental results concerning the problem of using the SMP observations for improving the catalogues were obtained by D.P.Duma. He showed that the accuracy of the α–zero point correction is better if the orbit of a minor planet is close to the Earth's orbit (Duma 1975). He also concluded (Duma 1995) that the non–precession motion of the equinox (Fricke 1980, 1982) was an artificial effect generated by the use of the non–correct value of the solar mean motion.

Essential progress in solving the problem was due to the Hipparcos program (ESA 1997), which resulted in compiling the Hipparcos catalogue of stars' positions. For stars with magnitudes less than 9 the median precision of their positions at epoch J1991.25 was equal to $0.77/0.64$ *mas*. During the period of the Hipparcos satellite activity, positional observations of 48 minor planets were obtained. Processing of these observations (Bange & Bec–Borsenberger 1997; Bange et al. 1998; Bougeard et al. 1997) determined the Hipparcos orientation parameters with greater accuracy compared to the ground–based observation processing results. However, the accuracy of the Hipparcos catalogue orientation parameters obtained in these papers proved to be less than the internal accuracy of the stellar positions. Some improvement can be reached by doing the proper phase correction. The reduction procedure and the systematic errors due to the phase effect have been discussed (Hestroffer et al. 1995), (Hestroffer 1998). The combined processing of the Hipparcos observations of the minor planets and the ground–based observations of (324) Bamberga and (2) Pallas (Hestroffer et al. 1998) showed the noticeable improvement of accuracy of the determined parameters. One conclusion is clear. At present, high quality ground–based observations are still necessary.

In the present paper we give the results of processing the Hipparcos observations in combination with the ground–based photographic observations made at Nikolaev to obtain the mutual orientation of the Hipparcos and the DE200/LE200 frames of reference. The offsets due to the phase effect are taken into account.

2. Nikolaev ground–based observations

During 1961–1995, 2329 photographic observations of 12 SMP (1, 2, 3, 4, 6, 7, 11, 18, 39, 40, 532, 704) were obtained at Nikolaev astronomical observatory (Ukraine) in the PPM catalogue system. The zonal astrograph (D = 150 mm, F = 2000 mm, field: $5° \times 5°$) was used. The ephemeris position of the observed SMP was in the centre of the plate. The reference stars chosen were close to the SMP position and surrounded it uniformly. All the plate measuring was done using the Ascorecord machine.

Some information of the observations is given in Table 1. The values of the unit weight errors, σ_0, are obtained when determining only the SMP orbits (the catalogue corrections were not considered) and are given in the last column of Table 1 (Batrakov et al. 1998b). These values are influenced mainly by the errors of measuring the SMP positions. The use of a more accurate catalogue will not lead to a drastic change of σ_0. It is of interest to note that greater internal accuracy is typical for the brighter SMP. These observations have already been used (Batrakov et al. 1998a, 1998b) to determine the orientation of the FK5 catalogue with respect to the dynamical frame of reference given by the DE200/LE200 ephemeris (Standish 1990) along with some periodic errors of the catalogue.

In these papers the equations of motion of the SMP in the rectangular frame of reference along with the equations in variation were integrated by the 15th order Everhart method (Everhart 1985). The perturbations due to the attraction of the 9 major planets were taken into account, and their positions were computed from the DE200/LE200 ephemeris. As the latter includes the perturbations from five minor planets (NN 1, 2, 4, 7, 324), the influence of these on the SMP motion was considered. The equations of motion included the terms corresponding to the case of motion of the probe particle in the Schwarzschild spherically symmetrical field of the Sun for the harmonical coordinates (Brumberg 1991). To obtain the solution of the condition equations the LSM was chosen. The rectangular coordinates and the velocity components of the SMP at the epoch t_0 = 1983 09 23.0 TDB (JD 2445600.5) were taken as the orbital parameters to be improved.

Let the catalogue frame of reference be defined by angles of rotation (counter-clockwise) around the x, y, z–axes of the dynamical frame of reference,

$$\epsilon_{xt} = \epsilon_x + \omega_x \,(t - t_0),$$
$$\epsilon_{yt} = \epsilon_y + \omega_y \,(t - t_0),$$
$$\epsilon_{zt} = \epsilon_z + \omega_z \,(t - t_0),$$

where $\epsilon_x, \epsilon_y, \epsilon_z$ are the initial values of the rotation angles and $\omega_x, \omega_y, \omega_z$ are the initial values of the angular velocities of rotation. All six are constants and are considered as parameters of rotation. For $\Delta\alpha = \alpha\,(cat) - \alpha\,(dyn), \Delta\delta = \delta\,(cat) - \delta\,(dyn)$ the following relations can be written:

$$\begin{aligned} cos\delta\Delta\alpha \;=\;& \;sin\delta\; cos\alpha\;(\epsilon_x + \omega_x(t - t_0)) \\ &+ sin\delta\; sin\alpha\;(\epsilon_y + \omega_y(t - t_0)) \\ &- cos\delta\;(\epsilon_z + \omega_z(t - t_0)), \qquad (1) \\ \Delta\delta \;=\;& -\Delta D - sin\alpha\;(\epsilon_x + \omega_x(t - t_0)) \end{aligned}$$

Table 1. The Nikolaev observations of selected minor planets

Asteroid number	Name of asteroid	Number of observations	Number of oppositions	σ_0
1	Ceres	211	21	0.15″
2	Pallas	258	25	0.17
3	Juno	243	24	0.18
4	Vesta	232	22	0.18
6	Hebe	219	22	0.18
7	Iris	193	19	0.20
11	Parthenope	191	19	0.20
18	Melpomene	209	19	0.19
39	Laetitia	237	24	0.20
40	Harmonia	203	21	0.21
532	Herculina	68	9	0.21
704	Interamnia	65	7	0.18
	Total	2329	232	

$$+cos\alpha\,(\epsilon_y + \omega_y(t-t_0)),$$

ΔD–correction being the constant systematic error of the catalogue declination system not connected to the rotation angles. When the classical way of determining the orientation through the Euler angles is used, the correction ΔA to the catalogue α–zero–point together with that of the Sun's mean longitude ΔL, counted along the ecliptic from the equinox for the data, and the inclination $\Delta\epsilon$ of the ecliptic to the equator are introduced. The parts of the condition equations depending on these angles are

$$\begin{aligned} cos\delta\Delta\alpha &= -(\Delta A + \Delta\dot{A}(t-t_0))\,cos\delta \\ &\quad +(\Delta L + \Delta\dot{L}(t-t_0))(cos\epsilon\,cos\delta \\ &\quad +sin\epsilon\,sin\delta\,sin\alpha) \\ &\quad -(\Delta\epsilon + \Delta\dot{\epsilon}(t-t_0))\,sin\delta\,cos\alpha, \\ \Delta\delta &= -\Delta D + (\Delta L + \Delta\dot{L}(t-t_0))\,sin\epsilon\,cos\alpha \\ &\quad +(\Delta\epsilon + \Delta\dot{\epsilon}\,(t-t_0))\,sin\alpha. \end{aligned} \tag{2}$$

Corrections of rotation parameters are connected to those of the Euler angles by

$$\begin{aligned} \epsilon_x &= -\Delta\epsilon, \epsilon_y = \Delta L\,sin\epsilon, \epsilon_z = \Delta A - \Delta L\,cos\epsilon, \\ \omega_x &= -\Delta\dot{\epsilon}, \omega_y = \Delta\dot{L}\,sin\epsilon, \omega_z = \Delta\dot{A} - \Delta\dot{L}\,cos\epsilon. \end{aligned} \tag{3}$$

In (2) and (3) dots indicate the time derivatives.

An additional effect included into the equations of condition is the correction for the photocentre offset. This effect had been studied often. The most recent studies deal with Hipparcos minor planet observations (Hestroffer et al. 1995; Hestroffer 1998). The corresponding formulae for usual $\alpha,\ \delta$–observations are found in Sveshnikov (1985) and partly in the Explanatory supplement for AENA (1961), and these are valid for a spherical asteroid having constant albedo over the surface. We give them below. The two laws of light scattering by the surface of the asteroid are considered: those of Lambert and Lommel–Seeliger. The corresponding distances between the photocentre and the geometric centre of the asteroid disk (offsets) respectively are

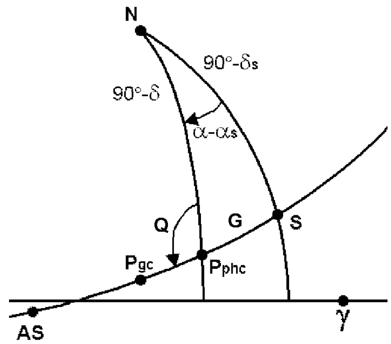

Fig. 1. Definition of values for taking into account the phase effect, the picture being given as seen from the Earth. S is the Sun, AS is the anti–Sun, P_{phc} is the photocentre of the asteroid disk, P_{gc} is the geometrical centre of the disk.

$$P_L(I) = \frac{3\pi}{16}\mu\frac{sinI\,(1+cosI)}{sinI + (\pi - I)\,cosI}, \tag{4}$$

$$P_{LS}(I) = \frac{2}{3\pi}\mu\frac{tg\frac{I}{2}\,[sinI + (\pi-I)\,cosI]}{1 - sin\frac{I}{2}\,tg\frac{I}{2}\,ln\,ctg\frac{I}{4}}. \tag{5}$$

I is the phase angle (angle between the Sun and the Earth as seen from the asteroid), and μ is the apparent angular radius of the asteroid disk on the celestial sphere. The corresponding corrections to the observation which reduce the latter to the geometrical centre of the asteroid disk are

$$cos\,\delta\Delta\alpha = p_\alpha,\ \Delta\delta = p_\delta, \tag{6}$$

where

$$p_\alpha = P(I)\,sinQ,\ p_\delta = P(I)\,cosQ. \tag{7}$$

For $P(I)$ one must take $P_L(I)$ or $P_{LS}(I)$ from (4) or (5) for one of the mentioned laws. Angle Q is the position angle of the anti–Sun centered at the photocentre of asteroid disk and counted over the celestial sphere counter–clockwise from the North Pole as seen from the Earth (see Fig. 1). It is determined (Explanatory supplement to AENA 1961) using the formulae:

$$\begin{aligned} cosG &= sin\delta_S\,sin\delta + cos\delta_S\,cos\delta\,cos(\alpha - \alpha_S), \\ sinG\,sinQ &= cos\delta_S\,sin(\alpha-\alpha_S), \\ sinG\,cosQ &= -sin\delta_S\,cos\delta + cos\delta_S\,sin\delta\,cos(\alpha-\alpha_S). \end{aligned} \tag{8}$$

Here G is the angular distance of the Sun from the asteroid as seen from the Earth, $\alpha_s,\ \delta_s$ are the Sun's spherical coordinates, and $\alpha,\ \delta$ are the observed coordinates of the asteroid photocentre. However we preferred to use relations

$$p_\alpha = kP(I)\,sinQ,\ p_\delta = kP(I)\,cosQ, \tag{9}$$

rather than (7). Here k is the non–dimensional coefficient introduced to compensate, partly, for the difference between the accepted law of scattering and the scattering of the real asteroid surface. These coefficients were considered as additional unknowns or were taken apriori. The results of processing (Batrakov et al. 1998b) were as follows (we put $k = 0$ for every of the 12 minor planets)

Table 2. Some FK5 zero–point determinations

Author	Object	Number of observ. Interval	ΔA (mas)	ΔD (mas)
Kolesnik 1995	Sun	15564 1968–1990	90 ±10	– 2 ± 9
	Mercury	3506 1960–1990	121 ±67	54 ±20
	Venus	11695 1960–1990	99 ±38	71 ±12
Branham and Sanguin 1996	21 minor planets	photo:4518 1909–1993	186 ±86	21 ±16
Poppe et al. 1998	Sun	9783 1974–1995	72 ± 5	88 ±16
Yagudina 1998	8 NEA	photo:1753 radar:118 1934–1995	126 ±57	57 ±37
Batrakov et al. 1998b	12 SMP	photo:2329 1961–1995	66 ±49	60 ± 5
Mignard and Froeschle 1997	Stars FK5 and Hip.cat.		–	∼60

$$\begin{aligned}
\epsilon_x &= 0.013 \pm .019'', \, \omega_x = 0.236 \pm .062''/cy, \\
\epsilon_y &= 0.015 \pm .020'', \, \omega_y = 0.004 \pm .066''/cy, \\
\epsilon_z &= 0.042 \pm .028'', \omega_z = 0.160 \pm .123''/cy, \\
\Delta D &= 0.059 \pm .004'', \\
\sigma_0 &= 0.18''.
\end{aligned} \tag{10}$$

When determining the classical corrections of the catalogue zero–points we put $\Delta\dot{\epsilon} = \Delta\dot{L} = 0$. Then the results were

$$\begin{aligned}
\Delta D &= 0.060 \pm .005'', \; \Delta L = 0.039 \pm .048'', \\
\Delta A &= 0.066 \pm .049'', \; \Delta\dot{A} = 0.068 \pm .107''/cy, \\
\Delta\epsilon &= 0.004 \pm .018'', \\
\sigma_0 &= 0.18''.
\end{aligned} \tag{11}$$

The unit weight errors in (10) and (11) give evidence of the good quality of the observations used. In Table 2 we give some results of the FK5 zero-point determinations. In the third column for every entry we give the number of observations (first line) and the interval covered by observations in years (second line). Our values of $\Delta A, \Delta D$ in Table 2 do not contradict the results of other researches.

After publishing the Hipparcos and Tycho catalogues (ESA 1997) we reduced the Nikolaev observations to the Hipparcos system, which is coincident with the ICRS within the errors of measurements (± 0.6 *mas* for each of the three axes). It has been proved that only about 60% of the stars required for reduction can be found in the Hipparcos catalogue; all other stars had to be taken from the Tycho one. Because the proper motions of Tycho stars have rather big errors, we have made three variants of the reduction:

Table 3. Parameters of orientation of the Hipparcos catalogue and their time changes for the three variants of observations' reduction (in *mas* for σ_0, ϵ and ΔD, *mas/year* for ω). The number of condition equations is 2329×2 = 4658. The number of unknowns is 12×6 + 7 = 79, N is the number of excluded condition equations. The epoch is JD 2445600.5

Variant of reduction	TYCHO		PPM		ACTRC	
ϵ_x	23.2	± 17.9	–12.1	± 17.0	8.0	± 16.0
ϵ_y	–13.5	18.5	–13.3	17.5	–20.0	16.0
ϵ_z	83.5	25.8	39.7	24.4	42.0	23.0
ω_x	1.5	0.6	2.4	0.6	1.3	0.5
ω_y	–0.7	0.6	–0.7	0.6	–0.4	0.6
ω_z	0.8	1.2	2.3	1.1	1.5	1.0
ΔD	–25.0	4.1	–25.1	3.8	–20.0	3.5
N	53		40		46	
σ_0	172		162		152	

1. Proper motions for the Tycho stars were taken from the Tycho catalogue;
2. These were taken from the PPM catalogue in which they are by order more accurate;
3. These were taken from ACTRC catalogue (Urban et al. 1998).

Reduction of observations to the ICRS system was realised using dependences for the three variants. The initial rotations, their velocities and ΔD–correction determined from observations for these variants are given in Table 3.

They give the orientation and motion of the ICRS with respect to the DE200/LE200 frame of reference at the epoch $1983\ 09\ 23.0$ TDB (JD 2445600.5). N in Table 3 gives the number of condition equations excluded during LSM process according to the 3σ–criterion. The phase effect was not accounted for. To correct for the systematic error of the Nikolaev row of the observations, ΔD was included. One possible cause of this error could be the neglect of the quadratic terms in the reduction formulae. The analogous error perhaps exists in the right ascensions as well but its detection is impossible due to correlation with ϵ_z. The third variant of the reduction proved to be the most accurate, and was used below.

In the final processing we included the phase effect offsets using Lommel–Seeliger low (5) and relations (9). The offset coefficients, k_1 (for Ceres) and k_4 (for Vesta) were determined by the LSM along with the orbital and catalogue parameters; for the rest of the minor planets we put $k = 1$. The initial number of equations of condition was 4658, number of unknowns was 12×6 + 9 = 81. The epoch of these parameters is $1991\ 07\ 20.0$ TDB (JD 2448439.5). The values of ϵ, ΔD, σ_0 are given in *mas*, the values of ω are in *mas/year*.

$$\begin{aligned}
\epsilon_x &= 19.3 \pm 17.3, \; \omega_x = 1.3 \pm 0.5, \\
\epsilon_y &= -19.8 \pm 17.9, \; \omega_y = -0.4 \pm 0.6, \\
\epsilon_z &= 41.8 \pm 26.0, \; \omega_z = 1.6 \pm 1.0, \\
\Delta D &= -20.1 \pm 3.5, \\
k_1 &= 0.88 \pm 0.35, \; k_4 = 1.25 \pm 0.45, \\
\sigma_0 &= 152.
\end{aligned} \tag{12}$$

Table 4. The ICRS orientation with respect to the dynamical frame of reference from the Hipparcos observations of minor planets (in *mas* for ϵ, *mas/year* for ω). N_e is the number of excluded asteroids.

Author	Bange & Bec-Borsenberger 1997		Bougeard et al. 1997	Bougeard et al. 1997	Bange et al. 1998	
Catalogue	FAST		FAST	NDAC	NDAC	
Number of asteroids	48		46	46	48	
N_e			(20, 27)	(20, 27)		
			c=0.1*	c=0.1*		
ϵ_x	3.71±2.00		1.5	3.9	3.4±2.2	
ϵ_y	–11.90	2.62	–12.3	–16.6	–12.2	2.9
ϵ_z	–12.64	4.12	–13.5	–5.1	–12.9	4.5
ω_x	4.32	1.18	3.8	2.4	4.2	1.3
ω_y	–9.51	1.47	–8.2	–10.2	–9.5	1.7
ω_z	14.91	3.68	13.1	16.7	16.3	3.9
					7.2	4.9**

* c is the tuning constant used in the algorithms based on Huber–c–regression

3. Observations of minor planets by the Hipparcos satellite

48 minor planets (their list contains 12 SMP observed in Nikolaev) were observed during 1989.85–1993.21 by the Hipparcos astrometric satellite. The procedure of obtaining and reducing these observations is described in (ESA 1997). The data obtained during this mission were analysed by the two teams of specialists, and two catalogues of the positional observations of minor planets were compiled: NDAC and FAST. The two catalogue data differ due to the way of processing the modulated light signal received from an observed asteroid. The difference is nonsensible for the minor planets having an apparent diameter of less than $0.1''$, the obtained positions in this case correspond to the photocentre. If the apparent diameter is more than $0.7''$ the FAST procedure is not adequate (ESA 1997). The observations are reduced to the geocentre and corrected for the stellar aberration and relativistic deflection of light due to the spherical gravitation field of the Sun. In these catalogues the corrections for the phase, shape and non–uniform albedo of the minor planets, were not taken into account. For every observation the following information is given: time moment, α_0 and δ_0 – the astrometric coordinates of the reference point, θ – the positional angle of motion of the slit. The data allow us to form the relation (ESA 1997)

$$\Delta s = (\alpha_o - \alpha_c)\, cos\delta\, sin\theta + (\delta_o - \delta_c)\, cos\theta, \tag{13}$$

which gives the corrections to the calculated position of the minor planet along the path of the slit motion. In short, one observation gives one equation of condition for Δs only. The general number of observations for the FAST catalogue is 2657 and for the NDAC it is 2837. The number of observations for the individual minor planets ranges from 12 to 123.

These observations were used for determining the Hipparcos catalogue orientation angles with respect to the dynamical one (Bange & Bec–Borsenberger 1997; Bange et al. 1998; Bougeard et al. 1997). In Bougeard et al. (1997) besides the classical LSM, robust methods were used to estimate the influence of the deflecting observations onto the results. The authors concluded that the LSM–solution is not stable with respect to exclusion of some minor planets from processing, and robust processing gives better results. Table 4 gives some results concerning the Hipparcos catalogue orientation parameters. In Bange et al. (1998) the epoch is JD 2448439.5, the mark $(**)$ in Table 4 means that the corresponding ϵ_z was obtained using 40 minor planets with angular diameter less than $0.15''$ to exclude the phase effect influence. In the remaining three mentioned papers the epoch was not indicated and one can surmise that the orientation parameters were given for the epoch of the Hipparcos catalogue, J1991.25.

4. Processing of the Hipparcos observations

When processing the Hipparcos observations as a model of motion of a minor planet we follow Batrakov et al. (1998b). The equations of motion of the minor planets in the rectangular frame of reference along with the equations in variation were integrated by the 15th order Everhart method (Everhart 1985). The major planet positions were computed from the DE200/LE200 ephemeris. As the latter includes the perturbations from five minor planets (NN 1, 2, 4, 7, 324), the influence of these on the minor planets was accounted for in our computations. The relativistic terms, as described in Sect. 2, were included in the equations of motion of the asteroids. The LSM was used to obtain the solution of the equations of condition. The epoch is JD 2448439.5.

At the beginning the elements of every minor planet were improved using the two catalogues, FAST and NDAC, separately. The erroneous observations were excluded according to the $3\,\sigma$–criterion. The maximum value of σ was 35 *mas* and the minimum one was 9.6 *mas* (no phase effect corrections were used) and 5.9 *mas* (phase corrections were added). The σ are greater for the minor planets with weak brightness, which agrees with Hestroffer results (1997). The same is true for the Nikolaev observations (see the last column of the Table 1).

Accounting for the phase effect during processing Hipparcos minor planet observations was discussed by Hestroffer et al. (1995) and Hestroffer (1998). The simplified model of Buratti & Veverka (1983) on the law of light scattering was used to account for the photocentre offset for the Hipparcos observations (Hestroffer 1998). The corresponding formula is

$$\Delta\, v = cos\,(\theta_s - \theta)\, C(i)\, sin\,(i/2)\, \phi/2, \tag{14}$$

where ϕ is the apparent diameter of a minor planet, i is the solar phase angle, θ_s is the position angle of the Sun in the tangent plane at the asteroid surface, θ is the position angle of the Hipparcos scanning direction **w**, the function $C(i)$ depends on the actual brightness distribution over the visible surface and it can be given with sufficient accuracy by the first terms of its series expansion: $C(i) = a + b\, i$. Because of a high correlation between the unknowns a and b only a single parameter, $\langle C(i)\rangle$, was determined (Hestroffer 1998), averaged over the phase angle.

Table 5. The phase effect. The $(C(i))$–values in the last column were obtained for the combined processing of the NDAC and FAST observations.

Minor planet	Catalogue of obser–vations	Without phase effect σ (*mas*)	Lambert law k σ (*mas*)	Lommel–S. law k σ (*mas*)	Buratti & Veverka law $(C(i))$ σ (*mas*)	Hestroffer 1998 $(C(i))$ σ (*mas*)
1	NDAC	10.4	0.70 8.1	0.80 8.1	0.57 8.2	0.63 9.6
	FAST	14.3	0.90 8.1	1.00 8.2	0.72 8.3	
2	NDAC	12.0	0.65 9.3	0.75 9.3	0.57 9.3	0.62 9.7
	FAST	12.8	0.85 9.8	0.90 9.8	0.70 9.8	
4	NDAC	10.5	0.75 6.6	0.80 6.5	0.56 6.5	0.75 7.2
	FAST	15.3	1.00 5.9	1.10 5.9	0.75 6.0	

Table 6. Results for the NDAC observations (in *mas* for σ_0 and ϵ, *mas/year* for ω, non–dimensional for k). Number of condition equations is 2837; for variant IV: 2837–182 = 2655. N_{un} is the number of unknowns. N is the number of excluded condition equations.

Variant	I	II	III	IV	V	VI
Number of asteroids	48	48	48	40	48	48
N_{un}	48×6+3	48×6+6	48×6+9	40×6+9	48×6+9	48×6+10
ϵ_x	1.1±1.7	2.6±1.7	2.5±1.7	4.8±1.2	2.2±1.7	2.3±1.7
ϵ_y	–10.8 2.2	–11.3 2.2	–11.3 2.2	–12.7 1.7	–12.2 2.2	–11.7 2.2
ϵ_z	– 0.8 3.5	0.2 3.6	– 0.9 3.5	– 2.1 2.0	0.1 3.5	–0.3 3.5
ω_x	–	2.4 1.0	1.4 1.0	2.4 0.9	0.8 1.0	1.1 1.0
ω_y	–	– 7.3 1.4	– 6.4 1.4	– 6.7 1.1	– 2.6 1.4	–4.9 1.6
ω_z	–	12.3 3.1	9.6 3.1	8.1 2.0	– 0.1 3.1	5.8 3.8
k_1	–	–	0.78 0.17	0.76 0.18	0.79 0.17	0.78 0.17
k_2	–	–	0.59 0.21	0.59 0.22	0.65 0.21	0.62 0.22
k_4	–	–	0.73 0.21	0.74 0.21	0.80 0.21	0.76 0.21
k_n	–	–	0.0	0.0	0.7	0.29 0.15
N	72	72	75	59	76	74
σ_0	15.2	15.2	15.0	14.1	15.1	15.0

In an attempt to choose the proper law of light scattering the offsets were computed with Lambert (4), Lommel–Seeliger (5) and Buratti & Veverka (1983) laws. We preferred in this section not to determine k or $(C(i))$ as an unknown through the LSM procedure but to take the fixed values for them. So, for every minor planet in Table 5 only 6 orbital parameters were determined for different accepted values of k. The smallest σ–value was the indicator for chosing the proper k or $(C(i))$. Table 5 gives k and $(C(i))$–values for which the observations are represented with the least σ. The last column of Table 5 gives $(C(i))$–values, obtained in (Hestroffer 1998) using observations of two catalogues, NDAC and FAST, together. Fig. 2 shows residuals with no phase effect correction (Part **a**), and with this correction (Part **b**) for (4) Vesta (FAST catalogue).

The data of Table 5 allow us to make the following remarks:

– The three laws of light scattering used give practically the same value of σ, though at different values of k and $(C(i))$. It is difficult to choose between them, and we preferred to use Lommel–Seeliger offsets in what follows.

– The NDAC and FAST catalogues require different values of k and $(C(i))$ for the same planet, the σ′s being practically equal. For the planets not covered by Table 5, having less angular diameters, σ is not changed if the phase effect is accounted for according to the one law of scattering or another.

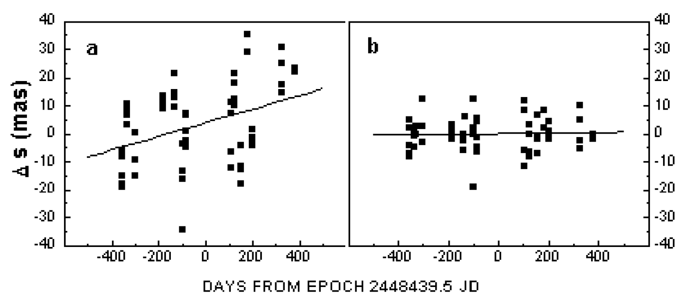

Fig. 2a and b. Residuals with no phase effect correction, **a**, and with phase effect corrections, **b**, for (4) Vesta (FAST catalogue).

Table 7. Results for the FAST observations (in *mas* for σ_0 and ϵ, *mas/year* for ω, non–dimensional for k). Number of condition equations is 2657; for variant V: 2657–167 = 2490. N_{un} is the number of unknowns. N is the number of excluded condition equations.

Variant	I	II	III	IV	V	VI
Number of asteroids	48	48	48	48 $(< 5\,\sigma)$	40	48
N_{un}	48×6+3	48×6+6	48×6+9	48×6+9	40×6+9	48×6+9
ϵ_x	0.1 ± 1.9	2.3±1.9	2.1±1.9	2.1±1.9	3.6±1.3	1.9±1.9
ϵ_y	–10.1 2.5	– 9.7 2.5	– 9.6 2.5	– 9.6 2.5	–13.0 1.6	–10.8 2.5
ϵ_z	– 4.6 3.9	– 4.8 3.9	– 6.4 3.8	– 6.4 3.8	– 0.7 1.9	– 5.0 3.8
ω_x	–	3.6 1.1	1.9 1.1	1.9 1.1	1.0 0.9	0.8 1.1
ω_y	–	– 9.2 1.5	– 7.4 1.5	– 7.4 1.6	– 2.9 1.1	– 1.6 1.5
ω_z	–	12.9 3.4	8.3 3.5	8.3 3.5	0.5 1.9	– 6.7 3.4
k_1	–	–	0.91 0.18	0.91 0.18	0.91 0.17	0.94 0.18
k_2	–	–	0.75 0.23	0.75 0.23	0.80 0.21	0.88 0.23
k_4	–	–	1.04 0.18	1.04 0.18	1.12 0.17	1.15 0.18
k_n	–	–	0.0	0.0	0.0	1.0
N	48	42	43	25	52	44
σ_0	16.2	16.1	15.8	15.9	14.7	15.8

– Without accounting for the phase effect the FAST observations of minor planets are less accurate than those of the NDAC.

In what follows the phase effect corrections were computed using the Lommel–Seeliger law, and observations of NDAC and FAST catalogues were processed separately. As noted earlier, the observation material is not uniform. There are two catalogues, the observations are grouped in series, and within these series the observations are apparently not independent. Moreover, the number of observations for minor planets, the intervals covering the observations, and the accuracy of the observations of asteroids are different.

In the present paper we have preferred to use the classical LSM because the robust methods are much more complicated and require troublesome computations. In order to get an impression of the influence of the deflecting observations, we have obtained the solutions for the cases when the observations were excluded from processing if their deflections exceeded 3σ or 5σ. The results are given in Table 7 (Variants III, IV) and they differ insignificantly. If there was no special notice, the 3σ–criterion was used. The observations within the catalogue were considered as having equal accuracy.

Several variants of the solution were obtained for every one of the catalogues. Processing was done for 48 minor planets as well as for 40 minor planets (6 minor planets having less than 30 observations, NN 12, 31, 51, 63, 196, 216, were excluded; asteroid N 27 was excluded due to abundance of deflecting observations; 33 observations of minor planet N 192 gave orbital parameters with too big errors). The solutions for the two catalogues are given in Tables 6 and 7. The phase coefficients for the minor planets NN 1, 2, 4 were included as unknowns into the LSM–processing. For the rest of the minor planets the $k = k_n$ was taken. In Variants III – V of Table 6 and in Variants III – VI of Table 7, k_n were taken fixed. In Variant VI of Table 6, k_n was included in the list of unknowns and its value was considered as a common one for remaining minor planets. The data given in Tables 6 and 7 allow us to draw the following conclusions:

– The difference of values of the parameters from one variant to another can be fully explained by their errors.
– The determined values of the rotation parameters depend sensibly on the phase effect and on the way it is accounted for. This determined dependence has been noted (Hestroffer et al. 1995). The phase effect influences mainly the determined values of ω_y, ω_z.
– k_1– and k_4– coefficients obtained from Nikolaev observations and given in (12), are in better agreement with the corresponding values obtained using the FAST catalogue (Table 7).
– Number of minor planets included in the processing influences the values of the determined parameters and their errors (Variants III and IV of Table 6, Variants III and V of Table 7).
– Discrepancies can be seen between values of some parameters, especially ϵ_z, obtained in this paper and by other authors (see Table 4) using the same Hipparcos observations and the same parameters under determination. Some discrepancies arise due to different integration, computing the differential coefficients and solving the normal equations.

5. The combined solution based on Hipparcos and Nikolaev observations

If the accuracy of the observations varies, a problem of proper weighting arises. We have accepted the standard weighting of condition equations according to the apriori errors of their right–hand parts. In general, for the Nikolaev observations we obtained $\sigma = 0.15''$. For Hipparcos we estimate $\sigma = 0.015''$–$0.020''$, excluding minor planet N 27 which has greater residuals. Therefore, for the weight of one Nikolaev observation we take $p = 0.01$, and for the Hipparcos ones $p = 1.0$. The observations of minor planet N 27 were excluded from subsequent

Table 8. Solutions based on the Hipparcos–NDAC and Nikolaev observations (in *mas* for σ_0 and ϵ, *mas/year* for ω, non–dimensional for k). The number of minor planets is 47. The number of condition equations is 2329×2+(2837–38)=7457. N_{un} is the number of unknowns. N is the number of excluded condition equations.

Variant	I	II	III	IV
N_{un}	47×6+3	47×6+6	47×6+9	47×6+9
ϵ_x	2.2±1.6	2.4±1.6	2.3±1.7	2.5±1.3
ϵ_y	–11.9 2.2	–11.5 2.2	–11.7 2.2	–12.7 2.2
ϵ_z	0.8 3.3	0.5 3.3	0.2 3.3	1.4 3.3
ω_x	–	0.4 0.3	0.4 0.3	0.4 0.3
ω_y	–	– 0.9 0.3	– 0.8 0.3	– 0.7 0.3
ω_z	–	– 0.4 0.6	– 0.6 0.6	– 0.9 0.6
k_1	–	–	0.72 0.15	0.71 0.15
k_2	–	–	0.43 0.20	0.43 0.20
k_4	–	–	0.90 0.18	0.91 0.18
k_n	–	–	0.0	0.7
N	111	111	113	113
σ_0	16.3	16.3	16.2	16.2

Table 9. Solutions based on the Hipparcos–FAST and Nikolaev observations (in *mas* for σ_0 and ϵ, *mas/year* for ω, non–dimensional for k). The number of minor planets is 47. The number of condition equations is 2329×2+(2657–35)=7280. N_{un} is the number of unknowns. N is the number of excluded condition equations.

Variant	I	II	III	IV
N_{un}	47×6+3	47×6+6	47×6+9	47×6+9
ϵ_x	2.6±1.7	2.8±1.7	3.0±1.7	3.8±1.7
ϵ_y	–10.2 2.1	– 9.8 2.1	– 10.1 2.1	–11.0 2.1
ϵ_z	– 4.0 3.2	– 4.2 3.2	– 4.6 3.2	– 3.6 3.2
ω_x	–	0.3 0.3	0.3 0.3	0.3 0.3
ω_y	–	– 0.9 0.3	– 0.8 0.3	– 0.6 0.3
ω_z	–	– 0.2 0.6	– 0.3 0.6	– 0.8 0.6
k_1	–	–	0.84 0.15	0.83 0.15
k_2	–	–	0.73 0.19	0.73 0.19
k_4	–	–	1.16 0.16	1.15 0.16
k_n	–	–	0.0	1.0
N	79	79	96	96
σ_0	15.5	15.5	15.4	15.4

considerations. The model of forces and the methods of computations were the same as in Sect. 4. The combined solution based on the Hipparcos and Nikolaev observations is given in Tables 8 and 9 for epoch JD 2448439.5. The following comments can be made concerning these solutions:

- Errors of the time changes of the rotation parameters were significally reduced after including the Nikolaev observations. Therefore, widening the observation interval, even at the expense of using ground–based observations, is inevitable if one wants to obtain sufficiently accurate values for changes of the rotation parameters.
- The estimated ϵ_z values are sensitive to the observational discrepancies between the FAST and the NDAC catalogues.

The comparison of our estimates with that of (Folkner et al. 1994), obtained using the VLBI and lunar laser ranging measurements is given in Table 10. The agreement of the results can be considered as satisfactory because the ϵ–values range in accordance with 3 σ–tolerances. As the epochs of the estimates differ, and the velocities of the rotation angles in later paper were not given, we had to transfer our results to epoch JD 2447435.5 (1 October 1988) which was taken from Standish et al. (1995).

Table 10. ICRS orientation with respect to DE200/LE200 (in *mas*). Epoch 1 October 1988.

Author	This paper Table 8,Var.IV	This paper Table 9,Var.IV	Folkner et al. 1994
ϵ_x	1.4 ±1.5	3.0 ±1.9	– 2 ±2
ϵ_y	–10.8 2.3	–9.4 2.3	–12 3
ϵ_z	3.9 3.7	–1.4 3.6	– 6 3

6. Conclusions

- The new processing of positional observations of 48 minor planets, obtained by the Hipparcos astrometric satellite, made in this paper shows that the velocities of changes of the rotation angles, ω, are sufficiently great, in accordance with those obtained by Bange & Bec–Borsenberger (1997); Bange et al. (1998); Bougeard et al. (1997).
- Combining the Hipparcos and the Nikolaev ground–based observations decreases the absolute values of velocities up to values which are comparable with their respective errors.
- The phase effect should be taken into account when processing accurate observations of minor planets, as already observed by Hestroffer et al. (1995); Hestroffer (1998).
- At the present state of the work, accurate ground–based observations of minor planets are quite useful in connecting the Hipparcos and DE200/LE200 frames of reference.

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References

Bange J.–F., Bec–Borsenberger A., 1997, In: Hipparcos Venice'97 Symposium. ESA SP–402, 169–172

Bange J.–F., Bec–Borsenberger A., Bougeard M.–L., 1998, In: Vondrák J., Capitaine N. (eds.) Journées 97. Systèmes de référence spatio–temporels. Prague, 22–24 Sept. 1997, 22

Batrakov Yu.V., Shor V.A., 1989, In: J.H.Lieske, Abalakin V.K. (eds.) Inertial Coordinate System on the Sky. 69–71

Batrakov Yu.V., Chernetenko Yu.A., Gorel' G.K., Gudkova L.A., 1998a, In: Vondrák J., Capitaine N. (eds.) Journées 97. Systèmes de référence spatio–temporels. Prague, 22–24 September 1997, 23

Batrakov Yu.V., Chernetenko Yu.A., Gorel' G.K., Gudkova L.A., 1998b, In: Contributions of Astronomical Society of Russia, 60–65 (in russian)

Batrakov Yu.V., Vashkevich A.S., Chernetenko Yu.A., Shor V.A., 1998c, Daily ephemerides of selected minor planets for 1999. Inst. Applied Astron., St.Petersburg, Russia, pp. 69

Bougeard M.L., Bange J.–F., Caquineau C., Bec–Borsenberger A., 1997, In: Hipparcos Venice'97 Symposium. ESA SP–402, 165–168

Branham R.L., Sanguin J.G., 1996, In: Lopez Garcia A., et al. (eds) Proceedings of the Third International Workshop on Positional Astronomy and Celestial Mechanics. Valencia. 429–435

Brouwer D., 1935, AJ 1022, 57–63

Brumberg V.A., 1991, Essential relativistic celestial mechanics. Alan Hilger, Bristol, Philadelphia, New York pp. 263

Buratti B., Veverka J., 1983, Icarus 55, 93

Dyson F.W., 1928, Transactions IAU 3, 227

Duma D.P., 1975, In: Modern problems of position astrometry. Nauka, Moscow, 105–107 (in russian)

Duma D.P., 1995, Kinematics and Physics of celestial bodies 11, No.6, 77–92 (in russian)

ESA, 1997. The Hipparcos and Tycho Catalogues. ESA SP–1200

Everhart E., 1985, In: Carusi A., Valsecchi G.B. (eds.) Dynamics of comets: their origin and evolution. Proceedings of IAU coll. 83, Reidel, Dordrecht, 185

Explanatory supplement to the Astronomical ephemeris and the American ephemeris and Nautical almanac. 1961, Her Majesty's Stat. Off., London, 533

Folkner W.M., Charlot P., Finger M.H., et al., 1994, A&A 287, 279

Fricke W., 1980, Celest. Mech. 22, 113–125

Fricke W., 1982, A&A 107, L13–L16

Hestroffer D., Morando B., Mignard F., Bec-Borsenberger A., 1995, A&A 304, 168–175

Hestroffer D., 1997, In: Hipparcos Venice'97 Symposium. ESA SP–402, 35–40

Hestroffer D., 1998, A&A 336, 776–781

Hestroffer D., Viateau B., Rapaport M., 1998, A&A 331, 1113–1118

Kolesnik Y.B., 1995, A&A 294, 874–894

Mignard F., Froeschle M., 1997, In: Hipparcos Venice'97 Symposium. ESA SP–402, 57–60

Newcomb S., 1895, The elements of the four inner planets and the fundamental constants of astronomy. Suppl. Amer. Eph., 1897, Washington, pp. 202

Numerov B.V., 1933, Bulletin de l'Institut Astronomique 32, 139–147 (in russian)

Numerov B.V., 1935a, AZh XII, 4, 339–348 (in russian)

Numerov B.V., 1935b, Comptes rendus de l'Academie des Sciences de l'URSS II, 451–454. (in russian) Comm.: Abridged version of (Numerov, 1935a)

Numerov B.V., 1935c, Journal des Observateurs XVIII, No.4, p. 57, Comm.: Translation of (Numerov, 1935a)

Numerov B.V., 1935d, AZh XII, 6, 584–585 (in russian)

Numerov B.V., 1936, AJ 45, 105–111

Orelskaya V.I., 1975, Astrometry and astrophysics No.26, Naukova Dumka, Kiev, 11–16 (in russian)

Orelskaya V.I., 1978, In: New ideas in astrometry. Contributions of 20–th astrometric conference (Poulkovo, 20–23 Mai 1975), Nauka, Leningrad, 25–28 (in russian)

Orelskaya V.I., 1980, Letters to AZh 6, 318–320 (in russian)

Pierce D., 1971, AJ 76, 177–181

Poppe P.C.R., Leister N.V., Laclare F., Delmas C., 1998, AJ 116, 2574–2582

Samoilova–Yakhontova N.S., 1955, In: Contributions of 11–th astrometric conference of the USSR (Poulkovo, 24–26 Mai 1954), 78–82 (in russian)

Standish E.M., 1990, A&A 233, 1, 252–271

Standish E.M., Newhall X.X., Williams J.G., Folkner W.M., 1995, JPL Planetary and Lunar Ephemerides DE403/LE403, JPL IOM 314.10–127, 1-15

Sveshnikov M.L., 1985, Trudy ITA, USSR, 19, 31–74 (in russian)

Urban S.E., Corbin T.E., Wycoff G.L., 1998, AJ 115, 2161–2166

Yagudina E.I., 1998, In: Lopez Garcia A., et al. (eds) Proceedings of the Fourth International Workshop on Positional Astronomy and Celestial Mechanics. Valencia. pp. 59–64