ASTROINFORMATICS: STATISTICAL PROPERTIES OF FUNCTIONS OF PARAMETERS OF THE STATISTICALLY OPTIMAL APPROXIMATIONS OF SIGNALS

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Correct error estimates are very important to estimate statistical significance of results and to prevent fake discoveries. Classical methods are to determine separate errors of separate coefficients. However, due to generally nonzero correlations between the deviations of the coefficients due to observational errors of the input data, the simplified version of the evaluation may cause drastic errors of the estimate of variance $\sigma^2[G(C_\alpha)]$ of the function G of m parameters of approximation C_α, $\alpha=1..m$. A correct expression is $\sigma^2[G(C_\alpha)]=\sigma_0^2\cdot\Sigma_{\alpha\beta=1}^m A_{\alpha\beta}^{-1}\ (\partial G/\partial C_\alpha)\cdot(\partial G/\partial C_\beta)$ where $A_{\alpha\beta}^{-1}$ is a matrix inverse to the matrix of normal equations $A_{\alpha\beta}$. A typical error is to use an abbreviated formula $\sigma^2[G(C_\alpha)]=\sigma_0^2\cdot\Sigma_{\alpha=1}^m A_{\alpha\alpha}^{-1}\ (\partial G/\partial C_\alpha)^2$, what is equal to set nondiagonal elements of $A_{\alpha\beta}^{-1}$ to zero

This simplification is generally not correct (see 1994OAP.....7...49A, 2020kdbd.book..191A). Here the fit is $x_C(t)=\Sigma_{\alpha=1}^m C_\alpha\cdot f_\alpha(t)$, where $f_\alpha(t)$ are the basic functions, and σ_0 is the unit weight error.

Particularly, for the approximation and it's derivatives of power s, $\sigma_0^2[x_C^{(s)}(t)]=\sigma_0^2\cdot\Sigma_{\alpha\beta=1}^m A_{\alpha\beta}^{-1}\cdot f_\alpha^{(s)}(t)\cdot f_\beta^{(s)}(t)$.

Other methods are based on artificial data sets based either on mixing deviations from the fit, or on generating (normally distributed) random “observational errors).

The “bootstrap” method is often used during recent decades.. It has systematically different error estimates.

For a large number of data, results are expected to converge asymptotically to that in the matrix. Examples are discussed.